Cold atom ballistics by coherent control

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Cold atom ballistics by coherent control

Linda E. Reichl and Benjamin P. Holder

Center for Complex Quantum Systems, University of Texas at Austin, Austin, TX 78712

Abstract.
We use the technique of Stimulated Raman adiabatic passage (STIRAP) to affect transport of ultracold atoms between two optical lattices in relative motion. We show, using Floquet analysis and degenerate perturbation theory, that the dynamics of atoms in a particular time-dependent optical lattice system can be reduced to the 3-level STIRAP model, allowing for a simple description of their coherent acceleration.

Keywords: Stimulated Raman adiabatic passage (STIRAP), optical lattice, coherent control
PACS: 42.50.Vk, 03.75.Be, 05.45.Mt

INTRODUCTION TO THE STIRAP MODEL

Stimulated Raman adiabatic passage (STIRAP) is a model system introduced by Kuklinski *et. al.* [1] to describe the experimental results of Gaubatz *et al* [2] and others in which an atomic or molecular population was transferred using two laser beams. Given three energy levels (E_a, E_b, E_c) with corresponding states $(|a\rangle, |b\rangle, |c\rangle)$, it was observed that an initial population in $|a\rangle$ subjected to a counter-intuitive pulse sequence (the "pump" laser, coupling levels E_a and E_b, is applied *after* the "stokes" laser, coupling levels E_b and E_c) will undergo a complete transfer to state $|c\rangle$. The model which describes this phenomenon can be derived by assuming a three level system with nonzero dipole couplings d_{ab} and d_{bc}. Making the rotating wave approximation, the Hamiltonian for this model, in the basis $(|a\rangle, |b\rangle, |c\rangle)$, has the form

$$H(t') = -\frac{\hbar}{2}\begin{pmatrix} 0 & W_1(t') & 0 \\ W_1(t') & -2\Delta & W_2(t') \\ 0 & W_2(t') & 0 \end{pmatrix}, \tag{1}$$

where W_1 and W_2 are the Rabi frequencies associated to the two transitions, Δ is the equal detuning of the two lasers from their corresponding transition frequencies, and t' is a parameter. It is easy to see that this matrix has an eigenvector, with eigenvalue zero, of the form

$$|1(t')\rangle = \cos\theta(t')|a\rangle - \sin\theta(t')|c\rangle\,, \tag{2}$$

where $\tan\theta(t') \equiv \frac{W_1(t')}{W_2(t')}$. The characteristic STIRAP transition ($|a\rangle \to |c\rangle$ as t' goes from $-\infty$ to $+\infty$) is therefore guaranteed under the conditions

$$\lim_{t'\to-\infty}\frac{W_1(t')}{W_2(t')} \to 0 \quad \text{and} \quad \lim_{t'\to+\infty}\frac{W_2(t')}{W_1(t')} \to 0\,. \tag{3}$$

The adiabatic behavior of this model system, using Gaussian dependence of the W_i on t', is displayed in Figure 1.

Although STIRAP was derived for a three level system in the electronic energy spectrum of atoms and molecules, its usefulness as a model quantum system is much broader. For example, a novel application of the model was achieved by Eckert *et. al* [3], who considering states $|a\rangle$, $|b\rangle$ and $|c\rangle$ to be the ground states of three neighboring optical microtraps. By varying their relative spatial separations, atoms could be coherently transported between the two farthest separated traps. Here, we describe how the STIRAP model can be applied to cold atoms trapped in an optical lattice, using three Floquet states of an effective Hamiltonian (see next section) and couplings induced by the modulation of the lattice amplitudes.

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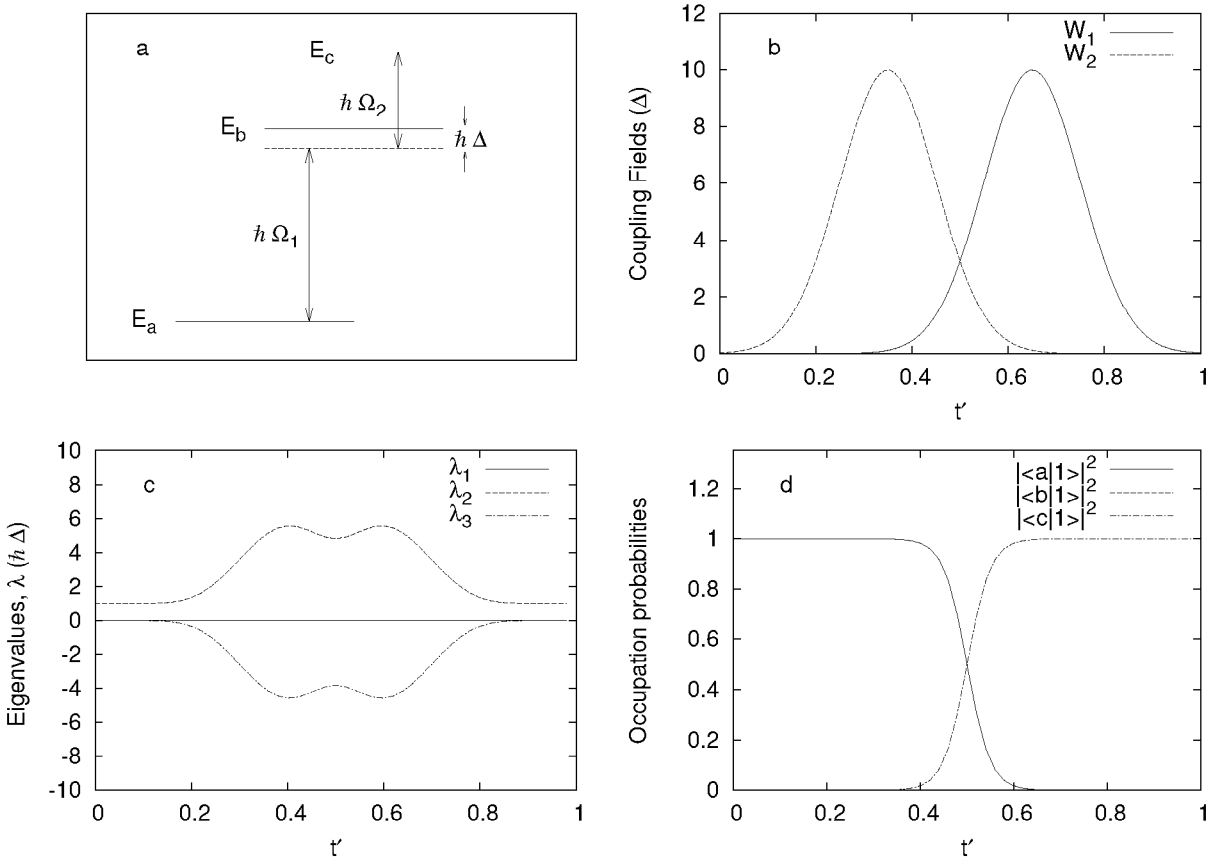

FIGURE 1. The 3-level "ladder" STIRAP system (Eq. (1)) with parameters $\Delta = 0.05$, $W_1 = W_2 = 1$ and $\hbar = 2$. Coupling fields are applied with frequencies equally detuned from the energy level spacings spacings by Δ (a). Adiabatic variation of the amplitudes of the coupling fields (b) in the manner described by Eq. (3) affects a transition within the $|1\rangle$ eigenvector from basis state $|a\rangle$ to $|c\rangle$ (d). The eigenvalue corresponding to this state remains unchanged at zero throughout the transition (c).

THE OPTICAL LATTICE EXPERIMENT AND ITS THEORETICAL DESCRIPTION

The cooling and trapping of neutral atoms with lasers relies primarily on the repeated interaction of laser photons with a single transition in the atomic energy structure. By manipulating this interaction with an atom's *internal* degrees of freedom, one can control the transfer of momentum to the atom and thus its *external* degrees of freedom, i.e. center of mass motion. Alkali atoms are some of the best candidates for cooling and trapping because the strong optical D_2 transition has a cycling transition in its hyper-fine structure, allowing for many photon absorptions without loss of population to other levels via spontaneous emission. In a typical experiment [4], millions of Cesium atoms are first cooled and trapped in a magneto-optical trap and then further cooled by the velocity-selective technique of Raman cooling [6] to obtain a dilute gas of atoms with sub-recoil velocities.

An optical lattice is created by subjecting the cloud of cold atoms to a pair of counter-propagating lasers, far-detuned from the atomic transition. The detuning of the lasers prohibits excitation of an individual atom but induces a spatially-periodic "light shift" of the ground-state energy. Theoretically, the Schrödinger equation for the dipole interaction of a classical standing wave of light (directed along the x-axis) with a two-level system can be reduced, via adiabatic elimination of the excited state dynamics, to a single equation for the center of mass dynamics of an atom in the ground state:

$$i\hbar\frac{\partial}{\partial t}|\psi_g(x,t)\rangle - \left[\frac{p_x^2}{2m} - \frac{d^2|E(x,t)|^2}{\hbar\Delta}\right]|\psi_g(x,t)\rangle\,. \tag{4}$$

The term in brackets on the right hand side of this equation is called the *effective Hamiltonian* where d is the dipole

matrix element coupling the two levels, Δ is the detuning of the laser frequency from the transition and $E(x,t)$ is the complex electric field amplitude [7, 8]. To the extent that the cold atoms can be considered non-interacting, the dynamics of the entire cloud in an optical lattice can be reduced to this simple one-particle equation. This will be the object of our study in the following sections.

A wide variety of effective Hamiltonians can be constructed through the use of multiple pairs of laser beams, variation of the relative frequencies of a given pair and modulation of the laser amplitudes. The stationary optical lattice, in the laboratory frame, is created by a single pair of lasers with equal frequencies yielding a complex electric field amplitude

$$|E(x,t)|^2 \sim \frac{E_0^2}{2}\cos(2k_L x)\,, \tag{5}$$

where E_0 is the real electric field amplitude and $k_L = \omega_L/c$ the wave number of the laser. Therefore the effective potential in Eq. (4) is proportional to a stationary cosine wave with half the periodicity of the laser light. The harmonic modulation of this stationary lattice amplitude ($E_0 \to E_0(t)$), creating an effective potential with three cosine terms, was used in the experimental discovery of "chaos-assisted tunneling" [5]. A pair of lasers with slightly offset frequencies $\omega_L \pm \delta\omega$ yields a "traveling" optical lattice in the laboratory frame, where

$$|E(x,t)|^2 \sim \frac{E_0^2}{2}\cos(2k_L x - \delta\omega t)\,. \tag{6}$$

The sum of Eqs. (5) and (6) allows for the "two-resonance" effective potential, which is the focus of our analysis in the following sections [9].

THE TWO-RESONANCE SYSTEM AND A STIRAP SUBSYSTEM

We will consider a "two-resonance" system, constructed by two pairs of counter propagating lasers: one pair with equal frequencies the second pair with slightly offset frequencies. The effective Hamiltonian for this system has the form

$$\hat{H}_0 = \hat{p}^2 + \kappa_0\left[\cos(\hat{x}) + \cos(\hat{x} - \omega_0 t)\right], \tag{7}$$

where all variables are in dimensionless units [from the units of the previous section: $p_x \to 2\hbar k_L p$, $x \to x/2k_L$, $t \to t/4\omega_r$, $\delta\omega \to 4\omega_r\omega_0$ and $(d^2E^2/2\hbar\Delta) \to -4\hbar\omega_r\kappa_0$ with recoil frequency $\omega_r \equiv \hbar k_L^2/2m \approx 13$kHz for cesium] and the effective Planck's constant is unity. Solving the Schrödinger Equation with a time-periodic Hamiltonian is accomplished by Floquet Theory, which guarantees solutions of the form

$$|\psi_\alpha(t)\rangle = \exp^{-i\bar{\varepsilon}_\alpha^0 t}\,|\bar{\phi}_\alpha^0(t)\rangle \quad \text{with} \quad |\bar{\phi}_\alpha^0(t+T_0)\rangle = |\bar{\phi}_\alpha^0(t)\rangle\,, \tag{8}$$

where $|\bar{\phi}_\alpha^0\rangle$ are called *Floquet eigenstates*, $\bar{\varepsilon}_\alpha^0$ are called the *Floquet eigenvalues* and $T_0 = 2\pi/\omega_0$. In the following, we will also assume that the solutions are spatially periodic with the period of the optical lattice. This can be approximately satisfied experimentally by the preparation of a cloud of atoms with sub-recoil velocities. Ten of the Floquet eigenvalues for this Hamiltonian, with $\omega_0 = 10/\gamma$ ($\gamma = 1.6180\ldots$ is the golden mean), are plotted as a function of κ_0 in Figure 2a. Associated Floquet eigenstates, at $\kappa_0 = 1$, are shown in their Husimi representation in Figure 3.

In order to accelerate trapped atoms, we will consider three Floquet states of the two-resonance system as a basis on which to perform a STIRAP-like transfer. We assume that atoms are initially prepared in the stationary cosine potential and that the state of an individual atom is well approximated by the Floquet state $|\bar{a}\rangle$ $\left(\equiv |\bar{\phi}_a^0(t)\rangle\right)$ shown in Figure 3. In order to transfer the atomic population to the state $|\bar{c}\rangle$, trapped in the traveling potential, a harmonic coupling of the two eigenvalues $\bar{\varepsilon}_a^0$ and $\bar{\varepsilon}_c^0$, through an intermediate eigenvalue $\bar{\varepsilon}_b^0$ is performed. By varying the strength of these couplings, a STIRAP transfer may be achieved. While classic STIRAP involves the dipole coupling of an atom's electronic eigenstates using a laser, the Floquet eigenvalues here can be coupled [8] by applying a small perturbation to the Hamiltonian in Eq. (7) of the form

$$\lambda\hat{V} = \lambda\cos(\hat{x})\left[\kappa_1(t')\cos(\Omega_1 t) + \kappa_2(t')\cos(\Omega_2 t)\right], \tag{9}$$

where the frequencies Ω_1 and Ω_2 are chosen to be equal (or nearly equal) to the eigenvalue spacings $|\bar{\varepsilon}_b^0 - \bar{\varepsilon}_a^0|$ and $|\bar{\varepsilon}_c^0 - \bar{\varepsilon}_b^0|$, respectively, at a particular value of κ_0; κ_1 and κ_2 will usually be Gaussian functions of the parameter t'.

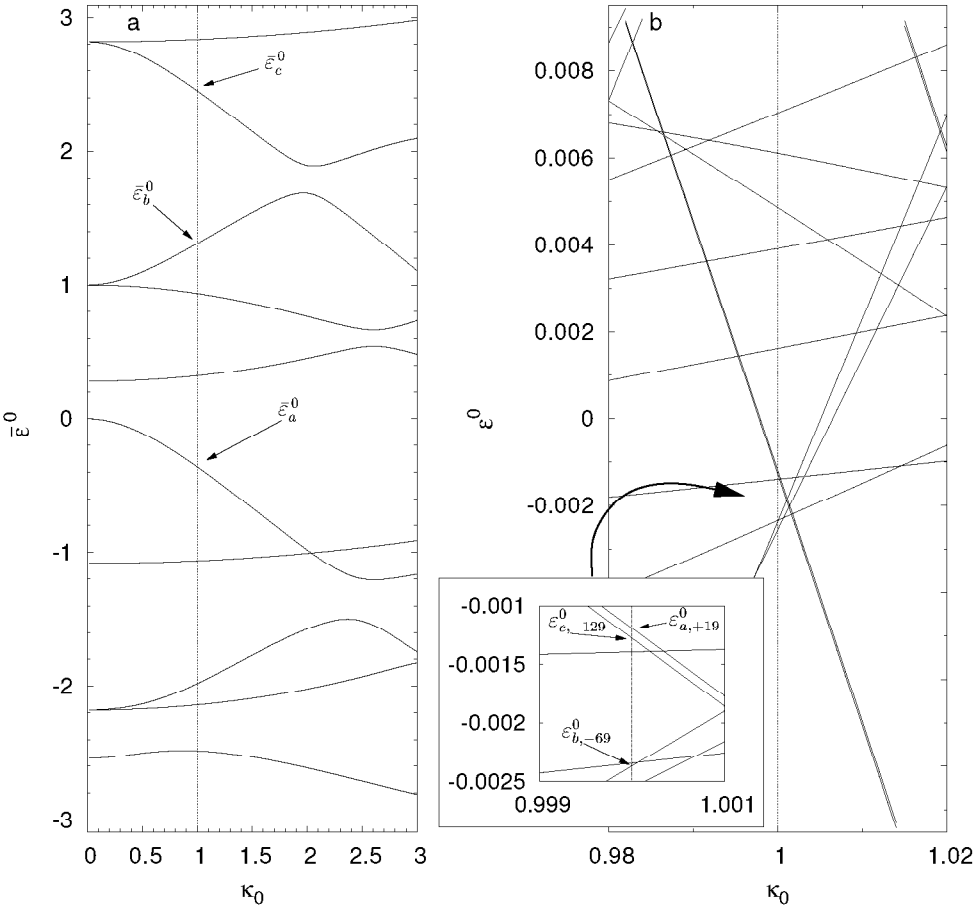

FIGURE 2. A few of the Floquet eigenvalues for the two-resonance Hamiltonian with $\omega_0 = 10/\gamma$ as a function of κ_0 (a). Figure (b) shows the corresponding "unbarred" eigenvalues in zone $-\omega/2 < \varepsilon^0 \leq \omega/2$ for the choice $(m_0, m_1, m_2) = (325, 88, 60)$ where $\omega_0 = m_0\omega$, $\Omega_1 = m_1\omega$, $\Omega_2 = m_2\omega$ and (Ω_1, Ω_2) have been chosen to approximately couple $(\bar{\varepsilon}^0_a, \bar{\varepsilon}^0_b)$ and $(\bar{\varepsilon}^0_b, \bar{\varepsilon}^0_c)$, respectively, at $\kappa_0 = 1$. The Q-indices of the unbarred eigenvalues (labeled in the inset) indicate the number of zones of width ω from the barred eigenvalue and satisfy $|Q_b - Q_a| = m_1$ and $|Q_c - Q_b| = m_2$.

Physically, this perturbation is achieved by modulating the stationary cosine potential amplitude about its κ_0 value (see Figure 4).

In order to see how the perturbation in Eq. (9) implies the existence of a STIRAP three-state model, we must consider a new Floquet analysis which applies to the full Hamiltonian $\hat{H} = \hat{H}_0 + \lambda\hat{V}$. To use Floquet theory, the frequencies ω_0, Ω_1 and Ω_2 must be chosen to be commensurate. Since ω_0 is assumed to be given, call it $\omega_0 = m_0\omega$, we must choose $\Omega_1 = m_1\omega$ and $\Omega_2 = m_2\omega$, with $m_i \in \mathbb{Z}$. Given a particular choice, the full Hamiltonian is periodic in time with period $T = 2\pi/\omega$ and solutions to the corresponding Schrödinger equation same have the form as in Eq. (8) except that the Floquet states now have period T, i.e.

$$|\psi_\alpha(t)\rangle = \exp^{-i\varepsilon_\alpha t}|\phi_\alpha(t)\rangle \quad \text{with} \quad |\phi_\alpha(t+T)\rangle = |\phi_\alpha(t)\rangle\,. \tag{10}$$

Note that we have written the Floquet solutions of the unperturbed system with an overbar and those of the full system without. When $\lambda = 0$, the two sets of physical solutions $(\{|\psi_\alpha(t)\rangle\})$ must be identical, however the Floquet solutions for $\hat{H}_0$ when considered a T-periodic system represent an infinite multiplicity of those when it is considered a T_0-periodic system. More precisely, for each barred Floquet state $|\bar{\phi}^0_\alpha\rangle$, there is a ($\lambda = 0$) unbarred state $|\phi^0_{\alpha,Q_\alpha}\rangle$ for every $Q_\alpha \in \mathbb{Z}$ corresponding to the same physical solution $|\psi_\alpha(t)\rangle$ [8]. The eigenvalues of these unbarred states occur in "zones" of width ω such that, in every zone, each barred eigenvalue $\bar{\varepsilon}^0_\alpha$ has a corresponding unbarred eigenvalue $\varepsilon^0_{\alpha,Q_\alpha} = \bar{\varepsilon}^0_\alpha + Q_\alpha\omega$. Since ω was chosen as the common factor in the frequencies Ω_1 and Ω_2 and those frequencies

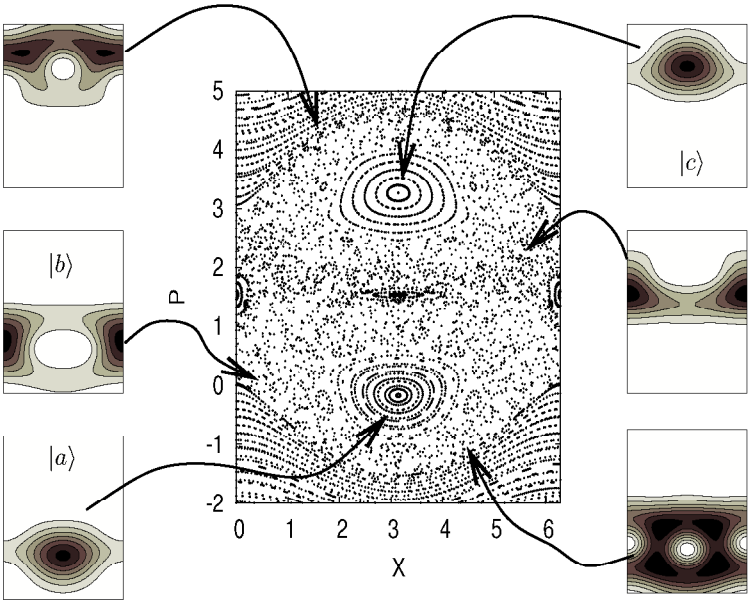

FIGURE 3. Husimi plots of the Floquet eigenstates (taken at $t = 0$) for the two-resonance Hamiltonian with $\omega_0 = 10/\gamma$ and $\kappa_0 = 1$. The association of each state to the classical phase space of the same Hamiltonian (strobed at the period $2\pi/\omega_0$) is shown with arrows (axes on the Husimi plots are the same as the classical phase space plot). Since these are time-independent representations of the Floquet states, the barred representation is identical to the unbarred (e.g. $|a\rangle = |\bar{a}\rangle$).

were chosen to couple the eigenvalues $\left(\bar{\varepsilon}_a^0, \bar{\varepsilon}_b^0, \bar{\varepsilon}_c^0\right)$, the unbarred eigenvalues $\left(\varepsilon_a^0, \varepsilon_b^0, \varepsilon_c^0\right)$ in a particular zone will be nearly degenerate. Figure 2b shows the near-degeneracy of the three unbarred eigenvalues corresponding to the states considered in the example above.

The perturbation analysis of a time-periodic system is identical to that of Rayleigh-Schrödinger (time-independent) perturbation theory when time is considered an additional *coordinate* in an extended Hilbert space [10, 11]. Therefore, following the usual degenerate perturbation procedure, the near-degeneracy of the three $\lambda = 0$ eigenvalues ε_{a,Q_a}^0, ε_{b,Q_b}^0 and ε_{c,Q_c}^0 requires that we write the zeroth-order solution $|\phi^{(0)}\rangle$ as a superposition of $|\phi_{a,Q_a}^0\rangle$, $|\phi_{b,Q_b}^0\rangle$ and $|\phi_{c,Q_c}^0\rangle$. Keeping terms to first order in λ, it can be shown that the perturbed system, in the limit of $\lambda \to 0$, reduces to an

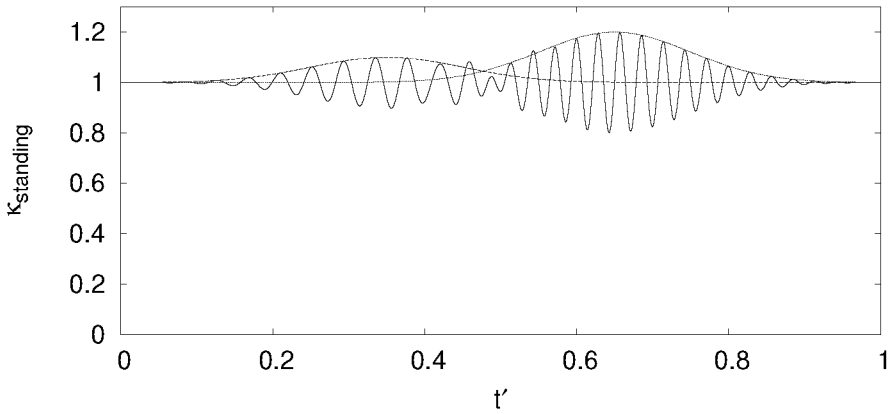

FIGURE 4. STIRAP transfer is performed by modulation of the stationary lattice amplitude (square of the electric field for one pair of lasers). Coupling of the two "upper" levels before the two lower levels, requires that the earlier perturbation has frequency Ω_2 and the later has frequency Ω_1. Here $\Omega_1 > \Omega_2$.

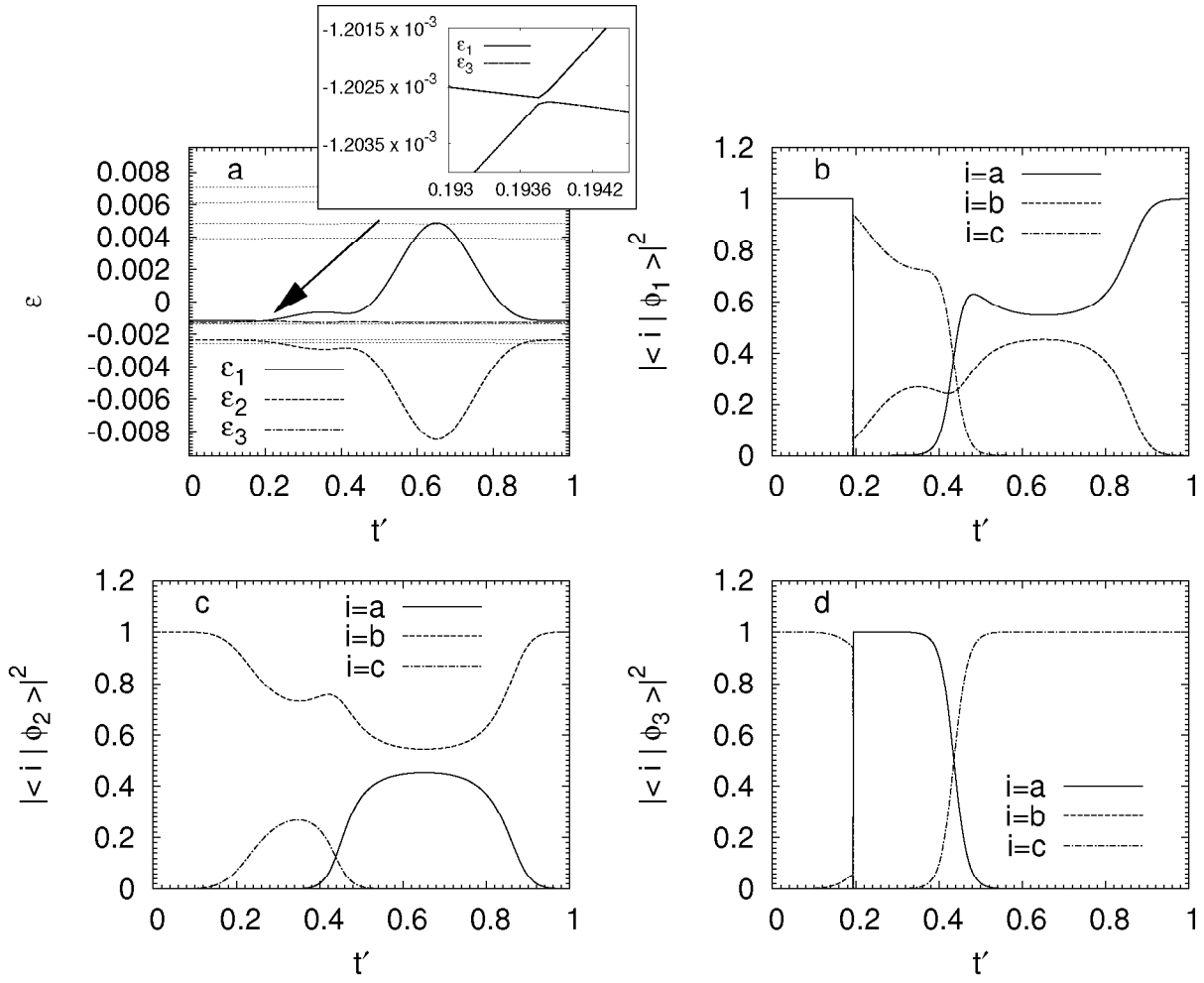

FIGURE 5. Floquet eigenvalues for the two-resonance system with applied perturbation $\hat{V}$ and $\lambda = 0.03$ are plotted as a function of the adiabatic time parameter t' (a). The three bold lines are eigenvalues ε_1, ε_2 and ε_3 (solid, dashed and dot-dashed, respectively) while the light dashed lines are the other eigenvalues shown in Figure 2 (avoided crossings between bold and light lines are vanishingly small and ignored). At $t' = 0$ and $t' = 1$, the eigenvalues are identical to those in Figure 2b at $\kappa_0 = 1$. The occupation probabilities in the basis $\{|a\rangle, |b\rangle, |c\rangle\}$ are plotted in (b)-(d), for states $|\phi_1\rangle$, $|\phi_2\rangle$ and $|\phi_3\rangle$ (initially equal to $|a\rangle, |b\rangle$ and $|c\rangle$, respectively). A narrow avoided crossing at $t' \approx 0.2$ (inset) prohibits the STIRAP transfer in the adiabatic limit [compare (b) and (d) to Figure 1(d)].

isolated three-level eigen-equation

$$\begin{pmatrix} 0 & \lambda\kappa_1(t')\,v_{ab} & 0 \\ \lambda\kappa_1(t')\,v_{b,a} & \Delta_1 & \lambda\kappa_2(t')\,v_{bc} \\ 0 & \lambda\kappa_2(t')\,v_{c,b} & \Delta_1 - \Delta_2 \end{pmatrix} \begin{pmatrix} C_a \\ C_b \\ C_c \end{pmatrix} = \varepsilon \begin{pmatrix} C_a \\ C_b \\ C_c \end{pmatrix}, \tag{11}$$

where $C_i = \langle \phi^0_{i,Q_i} | \phi^{(0)} \rangle$, $v_{ij} = \langle \bar{\phi}^0_i | \cos\hat{x} | \bar{\phi}^0_j \rangle / 2$, $\Delta_1 = \left(\bar{\varepsilon}^0_b - \bar{\varepsilon}^0_a\right) - \Omega_1$ and $\Delta_2 = \Omega_2 - \left(\bar{\varepsilon}^0_c - \bar{\varepsilon}^0_b\right)$. Apart from the unequal detunings Δ_1 and Δ_2 – which are a result to the requirement that the Hamiltonian $\hat{H}_0 + \lambda\hat{V}$ be time-periodic given an arbitrary ω_0 – this three level system is identical to the STIRAP model presented in Eq. (1). However, this slight difference destroys the adiabatic STIRAP transition when $\Delta_1, \Delta_2 \neq 0$ [8]. Figure 5 shows the adiabatic behavior of the example introduced in Figure 2 under parametrization

$$\kappa_i(t') = \exp\left[-\frac{(t' - t'_i)^2}{2(\sigma'_i)^2}\right], \tag{12}$$

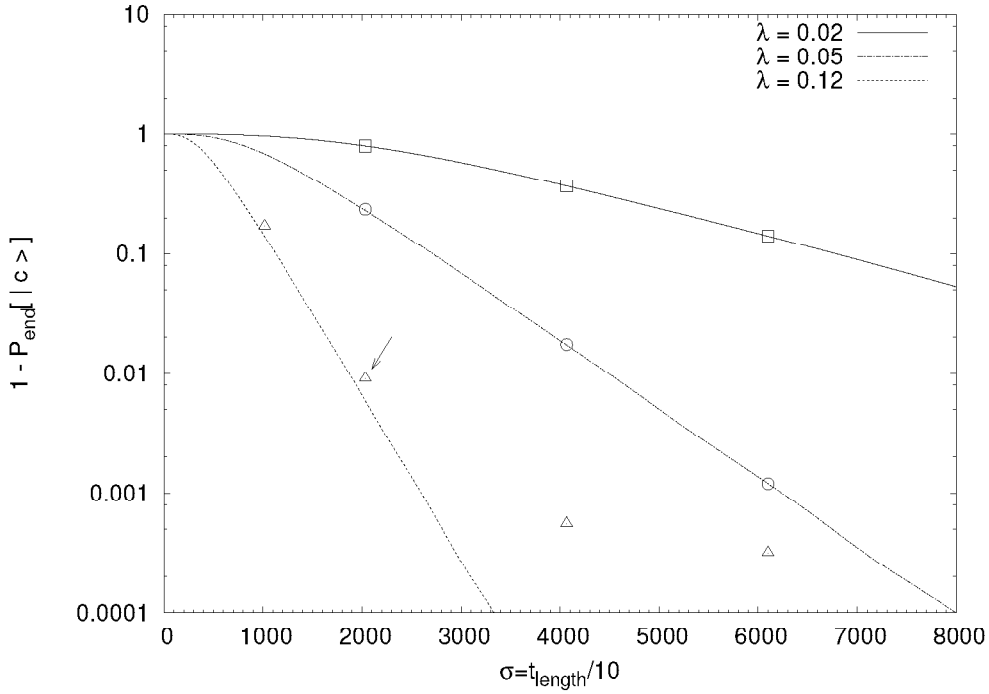

FIGURE 6. The approach to adiabaticity for the two-resonance system with resonant STIRAP coupling ($\Delta_1 = \Delta_2 = 0$). The probability of a non-adiabatic transition (transition from $|a\rangle$ to any state other than $|c\rangle$) is plotted as a function of σ, measured in dimensionless time. Lines are calculated using a three-level STIRAP model with matrix elements v_{ij} calculated from the Floquet eigenvectors associated to $\hat{H}_0$ with $\kappa_0 = 1$ and $\omega_0 = 10/\gamma$. The square, circle and triangle points are calculated by evolution of the Schrödinger equation for the full perturbed system, $\hat{H}_0 + \lambda\hat{V}$, for $\lambda = 0.02, 0.05$ and 0.12, respectively (Gaussian parameters here are those used in the adiabatic case of Figure 5, multiplied by t_{length}). The full time evolution of the point marked with an arrow is shown in Figure 7.

where $t_1' = 0.65$, $t_2' = 0.35$, $\sigma_1' = \sigma_2' = 0.1$ and the parameter t' varies between 0 and 1. A narrow avoided crossing between eigenvalues ε_1 and ε_3 (equal to ε_a and ε_c, respectively, at $t' = 0$) occurs at $t' \approx 0.2$ which induces an exchange of character between the two corresponding states $|\phi_1\rangle$ and $|\phi_3\rangle$. Therefore, in the truly adiabatic limit, atoms prepared in $|a\rangle$ at $t' = 0$ will remain in that state at $t' = 1$.

In the physical or numerical implementation of STIRAP transport for the two-resonance system, this problem can be avoided in two ways: one can apply resonant coupling frequencies ($\Delta_1 = \Delta_2 = 0$) or the evolution can proceed at a rate fast enough to pass through the avoided crossing. Considering the latter option, it is instructive to look at the Landau-Zener tunneling probability for the avoided crossing at $t' \approx 0.2$. This probability has the form

$$P_{\text{L-Z}} = \exp\left[\frac{-\pi\Delta_{\text{ac}}^2}{2\frac{dt'}{dt}\left|\frac{d}{dt'}(\varepsilon_1 - \varepsilon_3)\right|}\right], \tag{13}$$

where Δ_{ac} is the closest approach of eigenvalues ε_1 and ε_3 and $\frac{dt'}{dt} \equiv t_{\text{length}}^{-1}$ is the inverse of the non-adiabatic evolution time [12]. For the parameters used in Figure 5, the avoided crossing in the inset has $\Delta_{\text{ac}} = 1.05 \times 10^{-7}$ and $\frac{d}{dt'}(\varepsilon_1 - \varepsilon_3) = 2.58 \times 10^{-3}$ which gives an evolution time of $t_{\text{length}} = 1.5 \times 10^9$ for a 99% diabatic transition. Since this implies a physical time for Cesium atoms of about 8 hours (using ω_r and the non-dimensionalization given above), there is no danger of passing adiabatically through this avoided crossing in a real experiment.

ACCELERATION OF COLD ATOMS

In this section we consider the real-time evolution (numerically or in a laboratory) of the two-resonance system under the STIRAP coupling described above. An effective transfer of atoms involves a competition between two

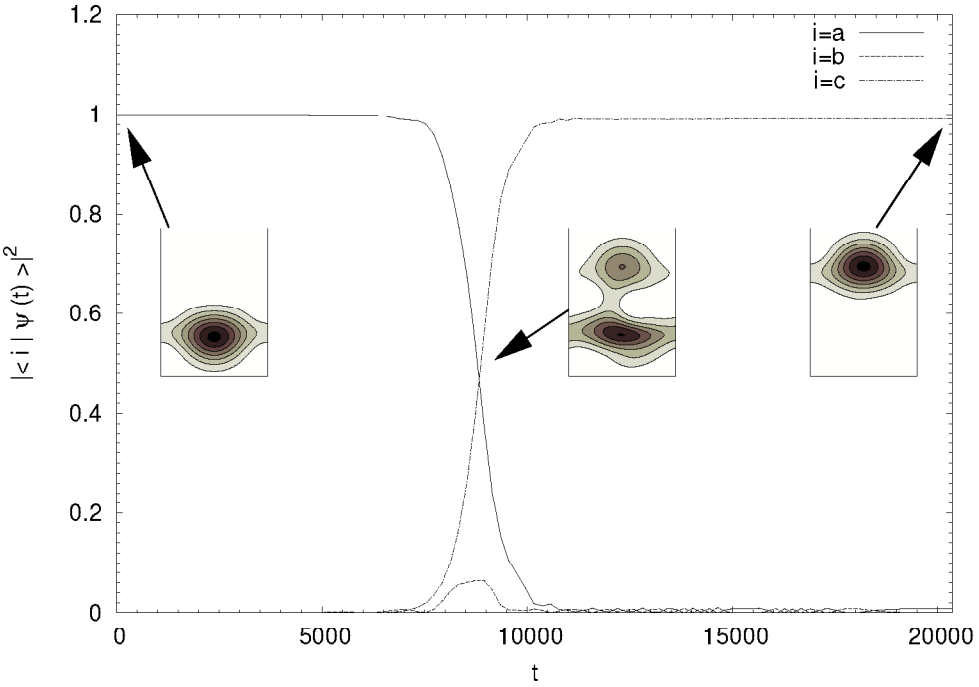

FIGURE 7. Time evolution of a state $|\psi(t)\rangle$ initially prepared in $|a\rangle$, under STIRAP coupling of the two resonance system with parameters $\kappa_0 - 1$, $\omega_0 - 10/\gamma$, $\lambda - 0.12$ (Gaussian parameters of the coupling fields are the same as in Figure 5). Husimi plots, with axes identical to those in Figure 3, are shown at three times.

factors. First, the STIRAP-like model in Eq. (11) is only an appropriate description of the system in the limit of small coupling, i.e. small λ. In the adiabatic analysis we have seen that there may be avoided crossings between the three relevant Floquet eigenvalues, which can impede the desired transfer. As λ is increased, these avoided crossings will grow larger – and others may appear – which will invalidate the simple STIRAP interpretation of the perturbed two-resonance system. The second factor is the necessity of a reasonable transfer time in the laboratory: population transfer must take place in less than a second (preferably quicker) in order for a given technique to be useful. This requirement is in competition with the first in that it requires that the applied couplings be relatively large, i.e. large values of λ.

The interplay of these two factors is evident in the graph shown in Figure 6. For smaller values of λ, the approach to adiabaticity in the full two-resonance system follows very closely that of the three-level STIRAP model, whereas it can be seen that the $\lambda = 0.12$ data points do not, particularly for long evolution times. As is expected, the transition of a state from $|a\rangle$ to $|c\rangle$ is not guaranteed for larger values of λ. The approach to adiabaticity itself is very similar to a Landau-Zener transition: the lines in Figure 6, for the evolution of the STIRAP model, clearly show an exponential approach. We have considered this evolution of the STIRAP model in Eq. (11) (with resonant, $\Delta_1 = \Delta_2 = 0$, coupling) over a wider range of parameters and found very good agreement with the empirical relationship

$$P_{\mathrm{na}} = \exp\left[-a\,\sigma\,\lambda\,\bar{v}\,f + 1.1\right] \tag{14}$$

where P_{na} is non-adiabatic transition probability; σ is the Gaussian width (in time) of both coupling fields κ_1 and κ_2; a is a constant of order unity, dependent on the overlap of the two Gaussian fields; $\bar{v} \equiv \left[v_{ab}^2 v_{bc}^2 / \sqrt{v_{ab}^2 + v_{bc}^2}\right]^{1/3}$; and $f \equiv \Gamma/(\mathrm{X} - 1 + \Gamma)$ with $\mathrm{X} \equiv \max\left(\frac{v_{ab}}{v_{bc}}, \frac{v_{bc}}{v_{ab}}\right)$ and $\Gamma \approx 100$. Therefore, we see that the desired transfer occurs more quickly (smaller σ) when the coupling strength λ is large and the coefficients v_{ab} and v_{bc} are nearly equal.

An example of the full STIRAP transition for the two-resonance system is shown in Figure 7. The evolution of the occupation probabilities is very similar to that of the STIRAP model (see Figure 1d) and the transfer efficiency here is better than 99%. Converting back to physical units, using the non-dimensionalization given above, the transfer time of $\sim 2 \times 10^4$ corresponds to about 0.4 seconds. Our chosen value of ω_0 implies that the momentum difference between the two optical lattices is $\Delta p \approx 3.3 = 3.3 \times 2\hbar k_L$, which for Cesium implies a change of velocity $\Delta \mathrm{v} \approx 2.3 \frac{\mathrm{cm}}{\mathrm{s}}$.

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REFERENCES

1. J.R. Kuklinski, U. Gaubatz, F.T. Hioe, and K. Bergmann. "Adiabatic population transfer in a three-level system driven by delayed laser pulses," Phys. Rev. A **40**, R6741 (1989).
2. U. Gaubatz, P. Rudecki, M. Becker, S. Schiemann, M. Külz, and K. Bergmann. "Population switching between vibrational levels in molecular beams," Chem. Phys. Lett. **149**, 463 (1988).
3. K. Eckert, M. Lewenstein, R. Corbalán, G. Birkl, W. Ertmer and J. Mompart. "Three-level atom optics via the tunneling interaction," Phys. Rev. A **70**, 023606 (2004).
4. D.A. Steck, W.H. Oskay, M.G. Raizen. "Observation of chaos-assisted tunneling between islands of stability," Science **293**, 274 (2001).
5. D.A. Steck, W.H. Oskay and M.G. Raizen. "Fluctuations and decoherence in chaos-assisted tunneling," Phys. Rev. Lett. **88** , 120406 (2002).
6. M. Kasevich, and S. Chu. "Laser Cooling below a Photon Recoil with Three-Level Atoms," Phys. Rev. Lett. **69**, 1741 (1992).
7. R. Graham, M. Schlautmann, P. Zoller. "Dynamical localization of atomic-beam deflection by a modulated standing light wave," Phys. Rev. A **45**, R19 (1992).
8. B.P. Holder and L.E. Reichl. "Stimulated-Raman-adiabatic-passage–like transitions in a harmonically modulated optical lattice," Phys Rev. A **76**, 013420 (2007).
9. Since the light shift potential is proportional to the square of the electric field amplitude, the two-resonance potential cannot be exactly achieved with two pairs of lasers (one pair having equal frequencies and the other with offset frequencies). However, the effect of unwanted cross terms created by the modulus squared of $E(x,t)$ for the four laser system can be minimized by an offset of the carrier frequency for the two pairs [8].
10. B.P. Holder and L.E. Reichl. "Avoided crossings in driven systems," Phys. Rev. A **72**, 043408 (2005).
11. H. Sambe. "Steady States and Quasienergies of a Quantum-Mechanical System in an Oscillating Field," Phys. Rev. A **7**, 2203 (1973).
12. M. Wilkinson and M.A. Morgan. "Nonadiabatic transitions in multilevel systems," Phys. Rev. A **61**, 62104 (2000).